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"Like-Me" Simulation as an Effective and Cognitively Plausible Basis for Social Robotics

William G. Kennedy · Magdalena D. Bugajska · Anthony M. Harrison · J. Gregory Trafton

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Abstract We present a successful design approach for social robotics based on a computational cognitive architecture and mental simulation. We discuss an approach to a Theory of Mind known as a "like-me" simulation in which the agent uses its own knowledge and capabilities as a model of another agent to predict that agent's actions. We present three examples of a "like-me" mental simulation in a social context implemented in the embodied version of the Adaptive Control of Thought-Rational (ACT-R) cognitive architecture, ACT-R/E (for ACT-R Embodied). Our examples show the efficacy of a simulation approach in modeling perspective taking (identifying another's left or right hand), teamwork (simulating a teammate for better team performance), and dominant-submissive social behavior (primate social experiments). We conclude with a discussion of the cognitive plausibility of this approach and our conclusions.

Keywords ACT-R · Theory of Mind · Embodied cognition · "Like-me" simulation · Cognitive plausibility

W.G. Kennedy (\boxtimes) · M.D. Bugajska · A.M. Harrison · I.G. Trafton

Naval Research Laboratory, 4555 Overlook Ave. SW,

Washington, DC 20375, USA e-mail: WKennedy@GMU.edu

M.D. Bugajska

e-mail: Magdalena.Bugajska@NRL.Navy.mil

A.M. Harrison

e-mail: Anthony.Harrison.ctr@NRL.Navy.mil

J.G. Trafton

e-mail: Greg.Trafton@NRL.Navy.mil

1 Introduction

How can we design and build social robots? We have previously suggested that if a system uses representations and processes analogous to those of humans, it will be able to interact more "naturally" with humans than one that does not [54]. Such a system will also be less likely to exhibit "alien" behaviors [10], those that lay outside social norms. This representational hypothesis prompted us to make use of findings from cognitive science research to guide the design of our robotic system and to implement the design using a cognitive architecture.

A cognitive architecture was defined by Allen Newell as a "fixed (or slowly varying) structure that forms the framework for the intermediate processes of cognition performance and learning" [36]. The concept is most recently defined as "a specification of the structure of the brain at the level of abstraction that explains how it achieves the function of the mind" [2, p. 7]. We based the design of our social robotic system on the computational cognitive architecture called ACT-R for Adaptive Control of Thought-Rational [2–4] rather than Markov random fields as discussed in the previous issue of this journal [12].

Most of ACT-R's history has been concerned with cognitive functions of the mind such as memory, problem solving, attention, and visual search [2, 4]. We extended ACT-R to control a physical robot (ACT-R/E, "E" for embodied). This required addressing the challenges of real sensor capabilities, localization, motion control, and the general noise inherent in real environments.

Cognitive science research tells us that successful modeling of social behavior, from imitative behavior to interpersonal communication, requires consideration of the knowledge, abilities, goals, and even feelings of others. The ability to infer that information and use it effectively is referred to



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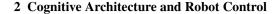
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Form Approved OMB No. 0704-0188 as having a Theory of Mind [42], perspective-taking [18] having a shared mental model [13], mind-reading [20], and "like-me" simulation [33]. This research points to the importance of mental simulation functionality within human cognition and led us to believe it should have a role in of our approach to social robotics.

Imitation, i.e., copying the actions of another, is proposed to lead to "an understanding of other minds" [32, p. 56]. Breazeal has used imitation for her robotics work on social robotics [7, 8]. Implementing and extending Meltzoff and Moore's theoretic model of how infants learn facial imitation [35], Breazeal and her team developed a humanoid robot called Leonardo that learns through imitative interactions and explores imitation as social interaction [9]. Their architecture employs a sensory system, a perception system, an action system, and a motor system that is comparable to ACT-R's modules, but their focus is on social interaction itself, and the communication facilitated by imitation, not the mental simulation nor cognitive plausibility on either side of the interaction. In contrast, we model the agent's ability to simulate another agent's decision-making by presuming the other agent is "like me" in physical and mental capabilities.

A "like-me" mental simulation capability is the ability to understand and predict the behavior of another based on your own capabilities. A "like-me" mirror neuron system has been hypothesized to explain the ability of infants, in the second half of their first year, to predict the goal of other people's actions and it has been hypothesized to be the basis of early social cognition [17]. There is also evidence that this capability is dependent on some learned knowledge. Falak-Ytter, et al. [17] also hypothesized that infants seem to require having learned a method of accomplishing a goal themselves before they can use their "like-me" simulation mechanism to recognize the same method in others, which it is why the capability is not seen until the second six months after birth. This simulation ability is the focus of this paper and our premise is that humans base their models of others on themselves, their own capabilities and knowledge, and by using a cognitively plausible system to provide this capability, we can build cognitively plausible, social robots.

The following sections give a brief overview of the ACT-R cognitive architecture as well as our embodiment of it on a robotic platform and its "like-me" simulation capability. We then present our work with models that incorporate mental simulation into reasoning about perspective taking, specifically handedness, simulation of another in support of efficient teamwork, and simulation of another to decide socially appropriate behavior in a competitive situation. We end with the discussion of cognitive plausibility of the mental simulation approach to social robotics and draw some conclusions.



ACT-R is a hybrid symbolic/sub-symbolic production-based system. Its modules are intended to represent specific cognitive faculties including declarative (fact-based) and procedural (rule-based) memory, visual and auditory perception, vocalization, manipulation, and time perception. Based on latest fMRI data, these cognitive faculties have anatomical correspondences [2]. The theory constrains the functionality and the integration of the modules in the actual computational implementation to facilitate cognitively plausible processing; notably, it enforces a serial memory access and execution and imposes bounds on the speed of processing of external inputs.

A cognitive model within the ACT-R architecture consists primarily of its initial declarative and procedural memories. The architecture is of a collection of functional modules, each of which exchanges information with the central procedural module through their respective buffer. During the execution of a model, ACT-R repeatedly: (1) matches the conditions of all productions against the current state of the buffers, (2) performs conflict resolution to select a single production to fire, and (3) as consequence of a production firing, changes the ACT-R state by modifying buffers' content or making a module request for an update. These updates can cause actions upon the world, e.g., grasping objects, navigation, etc., as well as internal state changes, e.g., changing intentions, deliberation, etc. The standard ACT-R's capabilities for interacting with the environment are limited to interacting with desktop computer environments.

ACT-R/E (for ACT-R Embodied), shown in Fig. 1, ventures beyond traditional computer displays and mouse/keyboard manipulation to establish embodied presence by first and foremost extending the representation of the visual and aural modules to enable 3D object and sound localization [28, 57]. We also extended ACT-R's capabilities to incorporate a locomotion faculty (the "moval" module) and a cognitive-map based spatial reasoning capability (the "spatial" module).

The "spatial" module maintains a representation of the space near the robot as a 2D map made up of rectangular cells but it does not maintain precise metric localizations for objects in those cells. The current cell location of the robot in this representation is maintained through interactions with the robot's "moval" module. To reason about objects observed in the environment, the module transforms each object's visual location coordinates into a cell in the internal map. The map is not available for reasoning directly, but on request to the spatial module, the module provides the identification of the closest object to the robot (or another reference agent) and the object's spatial relationship is in terms of cardinal headings, north, south, east, or west, from the robot or reference agent [27].



Fig. 1 ACT-R/E architecture (partially/fully shaded boxes are modifications/additions to ACT-R)

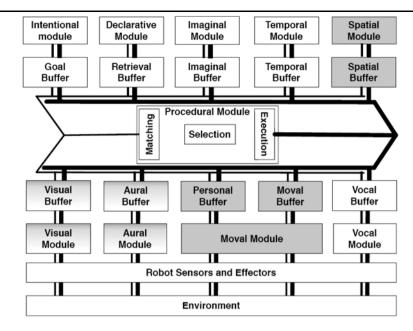


Table 1 Mapping of cognitive faculties to robot software

Cognitive faculty	Software	Hardware		
Locomotion	Localization, collision avoidance, path planning, and map learning [45]	Zero-turn-radius drive system Wheel encoders (odometers) Laser range finding system Sonars		
Vision	Person-tracking system [19] Color-blob detection [11]	Omni-directional camera High resolution forward facing camera Laser range finding system		
Hearing	Sound localization [31] IBM Via Voice TM (commercial speech recognition)	4-microphone sound array Wireless microphone (for the speaker)		
Speech	Cepstral Swift Text-to-Speech System (commercial speech generation)	Animated-lip synchronization Speakers		

Our robot, an iRobot B21r, is a human-scale, zero-turn-radius robotic platform best suited for use in indoor environments. The robot is equipped with an array of sensors and effectors including an animated face displayed on a robot-mounted LCD [39, 46], which allow it to perceive and interact with the environment. Table 1 shows our system-specific mapping of cognitive faculties to robotic software and corresponding sensors and effectors. The raw sensors' input, such as video and sound, are processed by the low-level robotic software and translated into feature-based, symbolic representations used by ACT-R/E modules as the data becomes available. Requests to the moval module in the form of relative or absolute motion-commands are passed onto our motion control subsystem, WAX [45]. Similarly, speech

module requests are forwarded to a commercial speech generation system, Cepstral. Finally, the animated face is synchronized with the speech output and indicates a change in the visual attention by turning to face the desired direction.

3 Mental Simulation and its Use as a Strategy

Mental simulation is incorporated in our models as part of the process of social reasoning. The overall modeling approach is based on cognitive science research: when humans are unsure of what to do, i.e., can not recall a previous instance of the current situation with previously decided action, they will try simulation to solve the problem [58].



Therefore, simulation can be viewed as a weak method to solve a problem [37]. In this section, we will discuss details of mental simulation itself and then how it is used as the foundation of a social strategy within our models.

Simulating the cognitive processes of another agent requires dealing with multiple states of reality. The best known examples are the problem spaces in Soar [30, 36] and alternate worlds in Polyscheme [14]. Soar's problem spaces facilitate automatic "subgoaling" and have been used to anticipate opponent's behavior in the game of Quake [29]. In the Quake system, Laird used some of the agent's own tactics to predict the tactics of the enemy in a manner similar to what we will describe more generally here. Polyscheme's worlds are a general construct and allow for instantiation and manipulation of hypothetical, counterfactual, and even stochastic simulations. The alternate worlds in Polyscheme have been used to build models of spatial perspective-taking [15] and a Theory of Mind [5]. ACT-R can build and maintain multiple representations of its world as well, to perform a cognitively plausible mental simulation of the decisionmaking of another agent.

We focus our paper around simulation in the spatial/embodied world, but the general mechanism we describe works in other domains as well. The "like-me" mental simulation uses the robot's own reasoning process to determine what it would do in the other agent's situation. The model prepares for this by effectively swapping places with the other agent: it creates a hypothetical or imagined representation of a transposed world in which it is located in the position of the other agent and the other agent is in its position with the information marked as imagined. To model some behaviors, additional transformations are necessary. To initiate the simulation, the model establishes a goal to determine the next action for the imagined situation. Its own productions then fire without modification to achieve this goal. Upon completion, the system has the next action for this imagined situation. It is a "like-me" simulation in that it uses it own capabilities, i.e., knowledge and reasoning process, to predict what the other agent will do.

The "like-me" mental simulation capability is implemented as the foundation of a strategy within our cognitive models. The simulation of another agent provides additional information used in the strategy to decide what action the agent will take. The design of such a model is shown in Fig. 2. The model has two strategies, an "individual" strategy and a "socially-aware" strategy. The "individual" strategy is the one the model would use to decide its next action without simulation. The "socially-aware" strategy is the one using a simulation of the other agent. When the simulation decides what the robot would do in the other agent's situation, the strategy uses that information to decide what it will do.

More formally, if an agent has a strategy, Π_i , that, in a context, c, determines its next action, A, as represented as:

$$\Pi_i(c) \Rightarrow A$$

then, using a "like-me" simulation and an appropriate transformation of the context c into c^t , the agent can have another strategy, Π_i , given by:

$$\Pi_i(c') \Rightarrow \Pi_i(\Pi_i(c^t) + c) \Rightarrow A'$$

where c' is the expansion of the original context to include the "like-me" prediction of the action of another agent in the transformed original context, c^{t} , and the process results in a potentially different action, A', based on using that knowledge.

This is a powerful mechanism because for any "individual" strategy the agent has, it can reason about another agent having that strategy and, further, by creating hypothetical situations (transposed or other transformation), it can predict the actions it would take under hypothetical conditions. Using that capability, the agent can change or adapt its own future actions, plans, and strategies with respect to the other agent. We will demonstrate some of this power and efficacy of this "like-me" mental simulation in three social scenarios beginning with a spatial perspective-taking task.

4 Cognitive Simulation of Spatial Perspective Taking

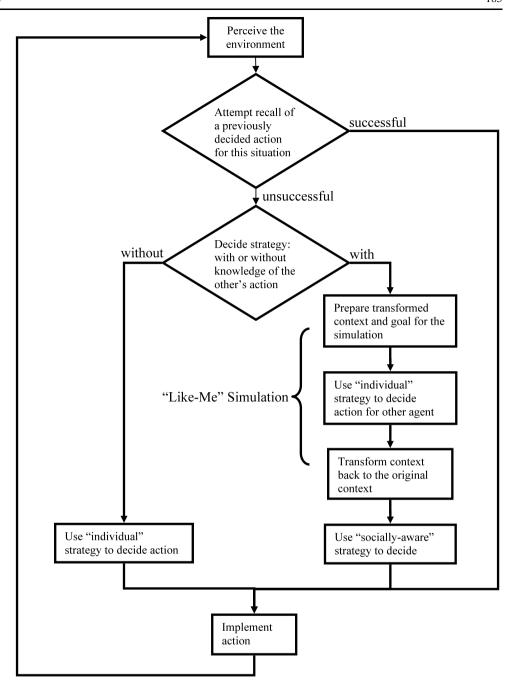
Spatial perspective taking is essential to the interpersonal communication associated with social robotics. Perspective taking aids in resolving spatial references [40] and is an assumed capability of other intelligent agents. In addition, the ability to resolve spatial references within different coordinate frames and to switch between reference frames is essential to efficient spatial task performance [60]. For example, spatial perspective taking is necessary to play "hide and seek" well [55, 56].

4.1 Perspective Taking Task

In our first scenario, the robot uses a "like-me" mental simulation to identify the spatial relation between entities in the environment. For this scenario, the space contained the robot, a person, and a box. The direction the person was facing was varied as was the location of the box with respect to the person., e.g., on the person's left, or on the person's right. The robot knows what those relationship terms mean with respect to itself, but not for the other person in this scenario. As a proof of concept, we simplified the problem to having the box placed to the east or west of the person and allowed the person to face one of four cardinal directions, north, south, east, or west. This provided eight possible problems that were generated randomly (with replacement) and four possible results: left, right, in front, and behind.



Fig. 2 Diagram of a cognitive model implementing "like-me" simulation



4.2 Cognitive Modeling Approach

The cognitive modeling approach is shown in Fig. 2. The ACT-R's basic reasoning process would first fire the compiled productions for the specific current situation, if any exist. However, when the model begins, it will not have such productions; they must be learned. Otherwise, the model attempts to recall a previously saved, declarative memory of a situation that matches the current situation. If successful, the response for that earlier situation is applied to this situation. If the attempted recall is unsuccessful, the model uses a

"like-me" simulation to perform the perspective taking. We started with the assumption that the robot knows the spatial relationship for an object near itself, either left of, right of, in front, or behind, as provided by our robot's spatial representation system. The model prepares for the "like-me" simulation by creating an imagined representation of the world with itself at the location of the other person by modifying its own location to be that of the person and marking the representation as imagined. It then establishes a goal for the simulation, to determine the relationship to the box. Its own productions then fire simulating the decision-making of the



person. Upon completion of the simulation, a new declarative fact is created for the specific situation and the response is saved for potential future use. Thus, the general pattern is using a simulation to learn an instance, then remembering that instance, and finally, with multiple uses, compiling that knowledge into a new production [26].

4.3 Perspective Taking Results

Our model was able to determine the relationship between the box and the person reliably and its performance improved with experience. It began by simulating novel situations, then retrieved previous solutions as the situation was repeated, and finally, it used learned new productions for specific situations. Each of these processes takes a different amount of time based on our cognitive model. (ACT-R predicts human performance times.) Our model accurately solved the perspective-taking problems and had three response times. Performing the mental simulation to determine the response takes 5.5 seconds in human terms. Using retrievals of previous situations takes an average of 3.8 seconds, and when productions were available, the model responds in 2.3 seconds.

4.4 Perspective Taking Discussion

In experiments testing human subjects in three groups by age (7–8, 12–13, and 18–22 years old), Ofte and Hugdahl [38] asked participants to perform a similar task (to identify a person's left or right hand in a variety of poses). Participants were timed as they responded. Our hypothesis is that with age the reliance on mental simulation for such a simple task is replaced with recall and then with fast, proceduralized knowledge. Figure 3 shows data from the original experiment with 95 percent confidence intervals (CI) and our model's performance comparing the human subject age groupings for the three different strategies: simulation (7–8 years), recall of previous situations (12–13 years), and using productions that encode previous experience (18–22 years).

The model results are consistent with the human subject data based on the model's predictions being within the 95 percent confidence intervals. The model's different mechanisms generate different response times, which align with the response times of the age groupings of the human subjects. This successful data match suggests that our implementation of "like-me" simulation with the associated learning process, is cognitively plausible and we can use this as strong encouragement for the plausibility of our mechanisms. It also provided the confidence to attempt a more significant task, developing a model of the behavior of a teammate.

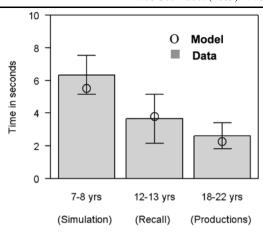


Fig. 3 Human and model data in perspective taking (with 95% CI)

5 Cognitive Simulation to Predict Another's Behavior

Since the "like-me" simulation capability, in general, and its implementation in ACT-R/E, has been shown to match human data and therefore have a degree of cognitively plausibility, we next turn to a more challenging functional demonstration of simulation, teamwork.

5.1 Teamwork Domain and Task

5.1.1 Teamwork: The AI Perspective

General models of teamwork and collaboration within the field of Artificial Intelligence (AI) include: STEAM and TEAMCORE [52], SharedPlans [21], COLLAGEN [43], and RETSINA [51]. Key issues in multi-agent systems research include the organization and make up of teams, task allocation among team members, multi-agent planning (including recognizing and resolving conflicts among agents and within plans), managing limited resources, communications among agents (including contingencies for when there is no communication), adaptation and learning in the team, and agent tracking and monitoring. For a broad overview of teamwork in multi-agent systems, see [47, 49, 50].

Of particular importance to our interests are systems that develop and use models of their teammates and then use that knowledge to improve collaboration. Kaminka et al. [24] presents a technique that allows one agent (a coach) to predict the future behavior of other agents (its own team and the opponent team) in order to coordinate activities by observing those agents and building a model of their behavior. Observations are translated into a time series of recognized atomic behaviors, and these into subsequences that characterize a team (although not necessarily a single agent). Other researchers investigated just how much monitoring of another agent is sufficient for an agent to be an effective teammate [23].



Our approach is to model the other agent in order to reduce the amount of monitoring that is required, and to do so in a cognitively plausible way by having the robotic team member perform a "like-me" mental simulation of the other agent.

5.1.2 Teamwork: The Psychology Perspective

There are many studies on what makes an effective team [13, 16, 32, 48]. This research addresses the same key issues as the AI research. Their research methodology also included examining how high and low performing teams accomplish team-related tasks. The results suggest that the knowledge employed by a good team member has three components [13]:

- (1) Knowledge of own capabilities [meta-knowledge],
- (2) Knowledge of the task, and
- (3) Knowledge about the capabilities of their teammates.

Most researchers have suggested that these three components are deeply inter-related; i.e., without any one of these, a person is not a good team member. The sharing of the necessary understanding among teammates is frequently referred to as having a shared mental model and has been suggested to be key to understanding team performance [13].

We believe the first two components are addressed if the agent is competent in individual components of the task. The third component, the knowledge of a teammate's capabilities, focusing on the teammate's cognitive processes can be addressed in a cognitive model using a "like-me" simulation.

5.1.3 Laboratory Scenario

As a test bed for our research, we created a task for a human-robot team that focuses on the need to simulate the decision-making of a teammate. The scenario takes place in a warehouse, where teammates can frequently see each other and move freely throughout the area. In this scenario, the robot and human are a security team charged with patrolling the area and responding to alarms. If, while the human and robot are separately patrolling inside the area, an alarm sounds, their task becomes "manning" the two widely separated guard stations as soon as possible.

To complete this task successfully, both team members must be inside different guard stations. If both go to the same guard station, there is a cost to the team's response time for one of them to then go to the other station. We did not include communication between team members during the response to keep the model simple. Such a constraint is often useful in team sports and military domains. For the robot, the task requires several capabilities. It must know the whereabouts of its teammate while it is patrolling to be prepared for the alarm. It must know the spatial locations of

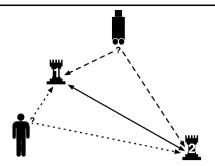


Fig. 4 Security team's alarm response decisions

the guard stations. It must be able to perform spatial reasoning to judge the distances to the guard stations (specifically which station is closer). Finally, the focus of this research, it needs to predict where the human will go based on the human's location. Figure 4 depicts the decisions for the two teammates.

5.2 Cognitive Modeling Approach

The robot's security guard behavior is made up of four distinct general strategies: patrolling the perimeter, maintaining spatial situation awareness, listening for the alarm, and responding to the alarm. To patrol the perimeter, the model uses the localization information obtained from ACT-R/E personal buffer, retrieves from memory the next waypoint for the current location, and then issues a command to the moval module to go to that point. To maintain spatial situation awareness during the patrolling task, the robot periodically locates its teammate, and updates its cognitive map. The spatial module continuously maintains the robot's own location automatically, but updates to the location of the teammate in the cognitive map are made only in response to explicit requests in productions and are based on visual information. While patrolling, ACT-R/E continuously monitors the aural buffer for the alarm event and, in response to an alarm detection, it initiates the alarm response strategy. The response strategy involves the robot choosing which security station to "man," retrieving its location from declarative memory, and issuing a command to the moval module to navigate to the recalled location. If the robot later detects its human teammate in the same guard station (a "conflict"), as part of the response strategy, it proceeds to the other guard station to complete the task. The conflict detection is based solely on short-range vision, but could have been designed to involve any form of perception or explicit communication.

We use the following two strategies for choosing which station to go based on the model diagram shown in Fig. 2:

(1) Individual strategy. An agent determines the station closest to its own position and goes there.



(2) Socially-aware (collaborative) strategy. An agent predicts its teammate's choice of destination and goes to the *other* station to avoid conflicts.

Both strategies are part of the robot's cognitive model and just like in the perspective-taking scenario, the robot creates and reasons about the hypothetical representation of the world with the position of the agents switched to determine its team member's destination. Once the robot concludes the simulation, it selects the other destination to avoid the conflict. The effect is that the robotic agent yields to what it believes will be the human's choice.

5.3 TeamBot Performance

We ran our ACT-R/E system both on a physical robot in our lab and in a desktop simulator to get performance data. In all runs, prior to the alarm, the robot kept track of the whereabouts of its teammate by periodically looking at the teammate every 10–15 seconds. This was indicated by a movement of the face shown on the robot's LCD monitor to turn and look at (attend to) the teammate. When the alarm sounded, the robot would respond using either the individual or the collaborative strategy depending on what was being tested.

5.3.1 The No Teamwork Case

The default case is that both the robot and human guards operate independently, i.e., both follow the individual strategy, and each would go to their closest guard station. In this case, their decision-making often results in both of them going to the same guard station, depending on the spatial situation at the time of the alarm.

Figure 5 is a trace of the locations of the human and robot during a run with each operating independently. The human began the run in the top of the diagram and the letters indicate the sequence of locations moving to the left: a, b, c, etc. The robot started at the bottom moving to the right. Both patrolled by moving counter-clockwise near the outer edges of the warehouse. At step "p" the alarm occurred and both started moving toward guard station #1 because it was closest to each of them at the time of the alarm. At time "u," the robot detected the conflict and started toward station #2. The conflict delayed completion of the alarm response task so that it ended at step "z."

5.3.2 The Teamwork Case

A trace of the second case is shown in Fig. 6. When the alarm occurred, the robot evaluated what the human would do by using a "like-me" simulation of the human's decision-making. It placed itself in the human's position and then used the same knowledge it would use to decide where to go,

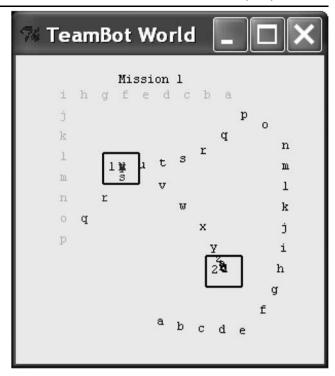


Fig. 5 Track of the human (*starting at top*) and the robot (*starting at the bottom*) "manning" two guard stations (*numbered squares*) with a conflict. Both began at their respective "a" locations and patrolled counter-clockwise until the alarm occurred at "p." Then both moved toward guard station #1 until, at step "u," the robot detects the conflict and goes to station #2

i.e., to the closest. Then knowing where the human would go, it immediately started toward the other guard station, thereby avoiding the conflict of both going to the same station. In this run, the team completed its task at step "x."

A series of desktop simulations of the scenario were run varying the starting positions of the robot and its human teammate along the top and bottom of the patrol area and with the human always going to the closest station, to put the human in the safest place and expose the robot to potential risk. The runs demonstrated that the performance with the robot simulating the decision-making of its human teammate was significantly faster in achieving the goal of "manning" both guard stations after the alarm. Specifically, with 25 simulated runs each, when the robot simulated the decision-making of its teammate, it took 3.28 fewer steps than the system that did not, t(27.7) = 8.1492, p < .001 with the Welch correction for unequal variances. Experimental results are reported using desktop simulations and the actual robot's behavior was very similar.

We also ran the same cognitive model on the iRobot B21r in our laboratory and demonstrated the successful embodiment of the system. A video of each case is available on our public website (www.nrl.navy.mil/aic/iss/aas/CognitiveRobots.php).



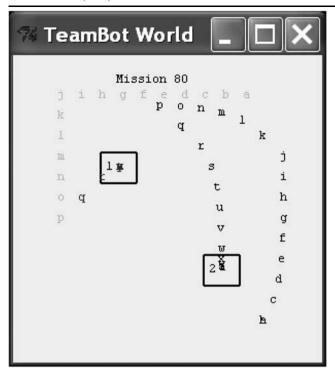


Fig. 6 Track of the human (starting at top) and the robot (starting at the bottom) avoiding a conflict by simulating the human's decision-making. Both began at their respective "a" locations and patrolled counter-clockwise until the alarm occurred at "p". The robot then, in accordance with the shared plan, determined where the human would go by simulating the human's decision-making using its own knowledge and therefore immediately started toward the other station

5.4 Discussion

The "like-me" simulation of a teammate within an embodied cognitive model facilitated the teamwork aspect of social robotics. Our robot reasoned about its teammate based on its own understanding of the situation and projected the decision-making of the human based on its own processes for appropriate teamwork behavior. The performance of the robot demonstrated that integrating a "like-me" simulation into a cognitive model can result in improved team performance.

The differences between the two test cases in the warehouse scenario are, of course, sensitive to the distances between the guard stations. We did not exhaustively test alternate configurations. In a more advanced scenario, communication between the team members or longer range detection of conflicts could reduce the difference between the two cases examined although in many teamwork situation, teammates do not communicate frequently (i.e., covert military operations).

While this model of teamwork allows us to demonstrate the concept and the implementation of the "like-me" simulation of the human teammate, it should be noted that the capability is more general than shown. Many other strategies can be accommodated because the agent's simulation is capable of applying to the simulated agent decision-making simulation any strategy the agent is capable of. For example, if the host is able to apply a game theoretic approach to the situation, it can simulate the other agent having that capability and act accordingly. However, there is a caution applicable here concerning Russell's Paradox [59], i.e., the difficulty of allowing a set to have itself as member. We must deal with the possibility that one of the strategies the other agent could have is to simulate the original agent, which could result in an infinite recursive loop. Similar to Russell's resolution of paradox, we did not allow this to occur by restricting the strategies that can be simulated to those that do not involve simulation. Additionally, the same simulation capability could be used to implement the cognitive equivalent to the minimax algorithm with total distance traveled by the team as the cost function, or any other strategy. The point of this discussion is that the simulation capability allows an agent to predict the reasoning of another agent based on its own capabilities and that is a very powerful mechanism. Although the teamwork model implicitly considered the social standing of team members, i.e., the robot yielded to the human, the next example addresses behavior of primates where social dominance drives the behavior to be modeled.

6 Cognitive Simulation to Model Social Dominance Behavior

Not long ago, a debate developed within the animal cognition community over whether chimpanzees are capable of perspective taking when competing for food in social situations. The debate was primarily centered on data and interpretations from two laboratories. Before the debate, most researchers agreed that chimpanzees had no perspective taking ability [41, 53].

However, in 2000, Hare et al. [22] suggested, based on their experiments, that chimpanzees do know what others can and can not see. Two years later, another laboratory reported that chimpanzees do not understand what others can and can not see, but use a variety of competitive strategies in social settings with competition for food [25]. This later study failed to replicate the results of the previous work. Five years after that and the most recently published work in this area [6], reasserted that chimpanzees really do have some perspective taking capabilities. The authors of the latest study observed that the size of the test area significantly affects the behavior of the chimpanzees and that they could explain why the intervening study failed to reproduce their results because it had used a significantly smaller test area.

We set out to model this social behavior using a cognitive model using a "like-me" simulation on our robot and focusing on only the relative social standing and spatial distances involved, not perspective taking or gaze following capabilities.



6.1 Social Dominance Task

Using different combinations of pairs of chimpanzees with known social dominance rankings, Bräuer et al. [6] reported the subordinate's behavior for food placed in two different locations which they hypothesized resulted in different "competitive intensities." The food was either placed in a "less competitive location" that was closer to the submissive or in a "competitive location" that was closer to the dominant. Additionally, the food was either hidden from the dominant behind an occlusion or visible to both chimpanzees. In the less competitive situation, they reported that the subordinate chimpanzee was able to reach the food quickly and chose the hidden or the visible food with equal likelihood (i.e., it did not take what the dominant could see into account). In the more competitive situation, the explanation was that the subordinate needed to take more time to get to the food and therefore went for the food that was hidden from the dominant more often than the food that was visible to the dominant. These results suggest that in the less competitive situation, the subordinate used a "grab and go" strategy because the dominant was less likely to be able to get to the food before the submissive, but in the more competitive situation, the submissive had to take what the dominant saw into account. In this experimental setup, the subordinate chimpanzee preferred to reach for the food hidden from the dominate chimpanzee. Note there was always a potential "cost" to the subordinate chimpanzee of getting too close to the dominant chimpanzee: physical punishment by the dominant. Also, note that dominant/submissive relations change depending on the pairs involved. We also assume that all chimpanzees have had experience as both a subordinate and dominant.

We endeavored to accurately replicate the scenarios and the effect of size of the test area reported by Bräuer et al.; however, we used a robot and a person as substitutes for the submissive and dominant chimpanzees. A diagram of the spatial layout of the two physical arrangements is provided in Fig. 7. The key features of the experimental setup were the fixed initial distance between the two chimpanzees, the relative distances of the food to each chimpanzee, and whether the food was observable by both chimpanzees or only the subordinate. Our system determined visibility based on spatial locations in the cognitive map, not based on geometric calculation of the line-of-sight for the agents, i.e., whether the food or a visually obstructing box was *closer* to the chimpanzee in question.

6.2 Cognitive Modeling Approach to Social Dominance

Our cognitive modeling of social dominance uses the same strategy selection approach as the previously discussed model of perspective taking (see Fig. 2). To decide what

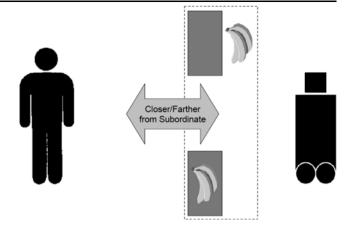


Fig. 7 Physical layout for social competition for food task

the robot acting as a subordinate would do, there were again three stages of learned behavior. First, if there are existing productions for the specific situation, they provide the response. If no productions are available, the model attempts to recall a previous decision for the same situation. If the retrieval is successful, the previous decision is applied. Failing that, the model uses a "like-me" simulation to decide what action to take.

The "like-me" simulation for this scenario is based on all chimpanzees having had experience as the dominant (the "individual" strategy) and that knowledge is used to inform the strategy as a subordinate (the "socially-aware" strategy). We started with creating procedural knowledge for what a dominant chimpanzee would do in various situations. The productions implement the "individual" strategy with the robot acting as a dominant chimpanzee and reaching for any observable, nearby food, or, if the food was not nearby or was not visible, ignoring the food, i.e., simply resting.

With the strategy for what to do as the dominant, the cognitive model for a subordinate chimpanzee includes the "socially-aware" strategy, which uses the "individual" strategy to determine what the dominant would do for the current situation using a "like-me" simulation. The first step is imagining being in the other chimpanzee's physical location and creating the goal to decide what action to take. The productions for dominant's "individual" strategy then fire to decide what the dominant chimpanzee would do based on the spatial information concerning the nearness and visibility of the food resulting from being in the dominant's location. With the determination of what the chimpanzee would do in the dominant's position, the model's "social-aware" strategy for the subordinate decides what it will do, either get the food or avoid conflict. It would not attempt to get any food that the dominant would reach for.

Tests of the system were conducted varying three factors. First was the position of the food relative to the two chimpanzees, i.e., closer to the dominant or closer to the subordinate chimpanzee. The second variation was whether the



dominant chimpanzee could see the food or only the subordinate chimpanzee could see it. Finally, we varied whether the robot was acting as the dominant or subordinate chimpanzee in the scenario. These factors supported running all variations of the cognitive model.

6.3 Social Dominance Results

Using the range of tests, we were able to replicate the basic results of the latest study [6] in a desktop simulated environment and on the physical robot in our lab. Acting as a dominant chimpanzee, the robot approached any visible and close food. As the subordinate chimpanzee, the robot preferentially selected the food that is hidden from the dominant chimpanzee when the food was closer to the dominant chimpanzee. It also went toward either the observable or the hidden food when it was closer when playing the subordinate chimpanzee as the real chimpanzees do. Videos demonstrating these behaviors are available on our public website, http://www.nrl.navy.mil/aic/iss/aas/CognitiveRobots.php.

6.4 Social Dominance Discussion

This scenario demonstrated several aspects of social robotics. First is perception, i.e., the ability to recognize another chimpanzee as another intelligent agent. The second aspect is the ability to perform spatial reasoning, which is used here to determine which chimpanzee is closer to the observable food. Finally, the scenario demonstrates the model's ability to predict the other's behavior in this social situation by using "like-me" mental simulation.

In the debate concerning whether chimpanzees know what another chimpanzee can see, our model can contribute to the discussion. In this case, a "like-me" simulation based on a chimpanzee's spatial reasoning capability, can explain the observed data. Besides being able to reason about which objects are closer to a chimpanzee in either the dominant's or subordinate's location, this model suggests that chimpanzees seem to understand that not everything they know is also known by the other. In setting up the simulation, besides swapping locations, we had to exclude from the simulated dominant's knowledge the location of hidden food, which the subordinate knew to allow the subordinate to decide to get that food. This modification of a simple, fully aware Theory of Mind was necessary get the model to match the data.

7 Cognitive Plausibility of "Like-Me" Simulation within ACT-R

The cognitive plausibility of a "like-me" simulation rests on its implementation, capabilities, and results. If the implementation included capabilities humans do not normally possess, there would be no possibility of the model's performance matching human performance data and the implementation would therefore not be considered cognitively plausible. The "like-me" simulation capability in humans has been seen in infants and was hypothesized to be an inherent capability [17, 33, 34].

The claim of the cognitive plausibility of using mental simulation to facilitate social robotics is based on several factors. First, the ACT-R architecture has a long history of successfully supporting models that compare well with both human cognitive functionality and psychological process data on a broad range of tasks, from basic problem solving and learning [1] to modeling car drivers [44]. Second, through the development of ACT-R/E, we have embodied ACT-R in the real world maintaining the theoretical precepts of the ACT-R theory. By building ACT-R/E interfaces to the external world using buffers and modules in a scientifically-principled way and consistently with the rest of the ACT-R family of theories, the ACT-R/E system is a highly coherent, integrated, and more cognitively plausible robotic architecture. Third, and most important, we believe our implementation of a general simulation capability based on ACT-R is justified because it maintains the core ACT-R theoretical precepts including serial production firing.

8 Conclusions

A "like-me" simulation is an approach to implementing a Theory of Mind and is a general but weak method for solving problems in social robotics. We have discussed the "likeme" mental simulation as a successful problem-solving approach, its implementation on a physical robot, its cognitive plausibility, and its demonstration in three areas of social robotics. It works by simulating the decision-making of another agent based on assuming the other agent has the same capabilities. We discussed the embodiment of the computational cognitive architecture ACT-R on a robot (ACT-R/E) in which we extended the capabilities of ACT-R to work in an embodied context while maintaining the design features that provide ACT-R its cognitive plausibility. Three uses of the "like-me" simulation approach in social robotics domains were presented: perspective taking, teamwork, and social dominance along with discussions of how well the model matched human and chimpanzee data, where available. Finally, the cognitive plausibility of this overall approach to social robotics was discussed.

The effort to model the perspective-taking problem of left-right determination was used as a proof of concept for a "like-me" simulation approach. We showed that a cognitive model that understands its left from its right and has the sensors to know the location and orientation of another agent can imagine itself in the other's position and then use



its own knowledge to determine the other's left from right. Using ACT-R's built-in memory and learning mechanisms, the model used simulation to solve the left-right problem in three stages: simulation, recall, and the use of compiled productions. The result of the spatial problem solving using the "like-me" simulation was saved in an instance-based representation for later recall and use. Over time, these instancebased representations were compiled into fast and computationally efficient representation (compiled productions) that solved the problem. The performance times for the different stages of problem solving, namely simulation, recall, and the use of compiled productions, compared well with experimental data on the performance of humans in different age ranges. This match of the model to human data at both a functional performance and reaction time level demonstrated the feasibility and cognitive plausibility of the "likeme" simulation approach for this simple problem domain.

The teamwork example applied the "like-me" simulation approach to a more complex, cooperative, social domain. Previous research suggested that a model of a competent team member needed to include an understanding of the capabilities of its teammate, i.e., having a shared mental model. Using a "like-me" simulation, our robot was able to predict the decision-making and future behavior of its teammate and act accordingly. The result saved time and steps and improved the performance of the team. This result added further support for the efficiency of simulation in an embodied context for social robotics.

The model of social dominance addressed reported results describing the competitive social behavior of chimpanzees. The model used its own productions for dominant behavior and the ability to apply the "like-me" simulation to determine how to behave as a subordinate. It did not need explicit productions for the behavior of a subordinate chimpanzee, only productions for the dominant's behavior and how to use that information to decide what to do as the subordinate. The "like-me" simulation of a dominant chimpanzee combined with filtered context information data provided the information necessary for the behavior of the modeled subordinate chimpanzees to match the behavior of the actual chimpanzees.

We acknowledge that the scenarios discussed could be more efficiently solved with specialized algorithms. However, there are problems with the use of specialized systems in the areas of re-usability, organization into an architecture, and scalability. Concerning re-usability, the restricted applicability of specialized algorithms leads to the need for many such algorithms to cover the range necessary for social robotics. The use of specialized algorithms, which are frequently incompatible, leads to their ad hoc organization into a robotic architecture. Finally, specialized algorithms do not scale well because of difficulties in how specialized algorithms are combined and integrated with other such systems.

We believe cognitive science has answers to these problems. Simulation addresses re-usability by re-using the agent's own knowledge to predict the behavior of the other agent without explicitly modeling the behavior of the other agent with specialized code. More broadly, models of individual behavior can be extended to model socially aware behavior by adding appropriate transformations and a "likeme" simulation capability. Instead of many specialized solutions organized in an ad hoc manner, mental simulation is implemented cleanly within the ACT-R/E cognitive architecture and is cognitively plausible based on using an architecture that has successfully matched human performance in a wide range of domains. Simulation is also a general mechanism in that anything the agent can do, it can model in another agent. Finally, with simulation integrated as part of a general strategy, the strategies based on simulation are already processed like other strategies in the cognitive architecture.

In this work, we explored the use of "like-me" simulation to achieve the functionality necessary for social robotics. This technique allows us to model/simulate anything the robot can do itself. As we build more models, we will naturally get more capabilities. These are just the beginning capabilities. Scaling up to tasks for which the robot's model does not currently do, should be feasible using first principles and general knowledge.

From this work, we draw several conclusions. We have shown that ACT-R can perform "like-me" simulations, preserving ACT-R's cognitive plausibility, as an effective approach to a Theory of Mind. Through the three examples, we have shown that "like-me" simulations can provide spatial perspective-taking functionality, contribute to teamwork modeling, and model social dominance behavior. The models of perspective-taking and chimpanzee social dominance were comparable to the experimental data and provided support for a theoretical basis based on a "like-me" simulation and spatial reasoning. Therefore, we suggest that "like-me" simulation in a cognitively-plausible architecture provides an effectively functional and cognitively plausible basis for social robotics.

9 Future Work

There are several avenues of future work based on the research described here. First, we plan to explore relaxing the "like-me" simulation assumption to allow minor modifications to the "like-me" capabilities. Our "like-me" simulation used the same rules under different situations. We would like to expand the use of rules to model slightly different capabilities, such as being able to move faster, see farther, and the like. Such changes would affect the teamwork simulation. Second, it would be nice to be able to use a "like-me"



simulation for capabilities that are very different than the subject agent, i.e., to be able to simulate what the agent can not actually do, but could imagine, such as flying. Third, we will explore other explanations for the social dominance behavior observed in non-human primates. This work is already underway. Finally, we would like to explore simulating or modeling other agents' potential behavior as it relates to decision-making for task assignments within a team, specifically, to be able to best assign tasks based on agents' different capabilities to optimize the team's performance.

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